

Chapter 2

Number Systems

SKEE1223 Digital Electronics

Mun'im/Arif

FKE, Universiti Teknologi Malaysia

September 24, 2015

Overview

- 1 Number Systems
- 2 Binary
 - Basics
 - Advanced
 - Exercises
- 3 Data Sizes
- 4 Hexadecimal
 - Basics
 - Advanced
- 5 Octal
 - Basics

Counting in Binary

- Binary mean two
- In binary number system, a digit has only two values: 0 and 1

| Decimal | Binary |
|---------|--------|
| 0 | 0 |
| 1 | 1 |
| 2 | 10 |
| 3 | 11 |
| 4 | 100 |
| 5 | 101 |
| 6 | 110 |
| 7 | 111 |
| 8 | 1000 |
| 9 | 1001 |
| 10 | 1010 |

Binary → Decimal

- Example: find the decimal value of 00101

| | | | | | | |
|---------------|----------------|----------------|----------------|----------------|----------------|-----|
| Bit Position: | 4 | 3 | 2 | 1 | 0 | |
| Binary: | 0 | 0 | 1 | 0 | 1 | |
| Decimal: | 0×2^4 | 0×2^3 | 1×2^2 | 0×2^1 | 1×2^0 | |
| | 0 + | 0 + | 4 + | 0 + | 1 | = 5 |

Therefore, $00101_2 = 5_{10}$

More Number Conversions

650₁₀ to binary using repeated division method:

| Value | Quotient | Remainder |
|---------|----------|-----------|
| 650 ÷ 2 | 325 | 0 |
| 325 ÷ 2 | 162 | 1 |
| 162 ÷ 2 | 81 | 0 |
| 81 ÷ 2 | 40 | 1 |
| 40 ÷ 2 | 20 | 0 |
| 20 ÷ 2 | 10 | 0 |
| 10 ÷ 2 | 5 | 0 |
| 5 ÷ 2 | 2 | 1 |
| 2 ÷ 2 | 1 | 0 |
| 1 ÷ 2 | 0 | 1 |

■ Therefore, 650₁₀ = 1010001010₂

Binary \longleftrightarrow Decimal

- Convert these binary numbers to decimal:
 - $1010_2 \Rightarrow 2^3 + 2^1 = 10$
 - $10111_2 \Rightarrow 2^4 + 2^2 + 2^1 + 2^0 = 23$
- Convert these decimal numbers to binary:
 - $19 = 2^4 + 2^1 + 2^0 = 10011_2$
 - $58 = 2^5 + 2^4 + 2^3 + 2^1 = 111010_2$

Common Bit Groups

| Name | Number of bits |
|--------------------|----------------|
| Nibble (or nybble) | 4 |
| Byte | 8 |
| Word | 16 or 32* |

* depends on system

- Word size is determined by manufacturer.
 - For computers (Intel processors), a word is 16 bits.
 - For handphones (ARM processors), a word is 32 bits.
 - There are processors that use word size $\neq 2^n$

Maximum Values

| Number of bits n | Largest value 2^{n-1} |
|-----------------------|----------------------------|
| 1 | 1 |
| 2 | 3 |
| 3 | 7 |
| 4 | 15 |
| 5 | 31 |
| 6 | 63 |
| 7 | 127 |
| 8 | 255 |
| 10 | 1023 |
| 12 | 4095 |
| 16 | 65535 |
| 24 | 16777216 |
| 32 | 4294967295 |

Multiples of Bit/Bytes

| Prefix | Symbol | Power-of-2 | Power-of-10 |
|--------|--------|------------|-------------|
| kilo- | K | 2^{10} | 10^3 |
| mega- | M | 2^{20} | 10^6 |
| giga- | G | 2^{30} | 10^9 |
| tera- | T | 2^{40} | 10^{12} |
| peta- | P | 2^{50} | 10^{15} |
| exa- | E | 2^{60} | 10^{18} |
| zeta- | Z | 2^{70} | 10^{21} |
| yotta- | Y | 2^{80} | 10^{24} |

Counting in Hexadecimal

- Hexadecimal (hex) means sixteen
- In hex system, a digit can take the value 0 through 9, A through F.
- Hex is used to write binary numbers in compact form.

| Binary | Decimal | Hexadecimal | Binary | Decimal | Hexadecimal |
|--------|---------|-------------|--------|---------|-------------|
| 0000 | 0 | 0 | 1000 | 8 | 8 |
| 0001 | 1 | 1 | 1001 | 9 | 9 |
| 0010 | 2 | 2 | 1010 | 10 | A |
| 0011 | 3 | 3 | 1011 | 11 | B |
| 0100 | 4 | 4 | 1100 | 12 | C |
| 0101 | 5 | 5 | 1101 | 13 | D |
| 0110 | 6 | 6 | 1110 | 14 | E |
| 0111 | 7 | 7 | 1111 | 15 | F |

Hexadecimal → Decimal

- Hexadecimal number conversion: Convert 1011011011001_2 to hexadecimal

Binary: 1011011011001

break binary into 4 groups

1 6 D 9

Hexadecimal: $16D9_{16}$

Hexadecimal \longleftrightarrow Decimal

- Convert the following to binary:
 - $CF8E_{16} \Rightarrow 1100\ 1111\ 1000\ 1110_2$
 - $974_{16} \Rightarrow 1001\ 0111\ 0100_2$
- Convert the following to hexadecimal
 - $1111\ 0000\ 1010_2 \Rightarrow F0A_{16}$
 - $10\ 0001\ 1101\ 1001_2 \Rightarrow 21D9_{16}$

More Number Conversions

- Convert $A7B_{16}$ to binary and decimal – easy
- Convert 650_{10} to hexadecimal – 2 ways
 - Convert to binary first, then to hex
 - Convert directly to hex

More Number Conversions

- 650_{10} to hexadecimal using repeated division method:
- $650/16 = 40.625$, $0.625 \times 16 = 10=A$ (Least significant bit (MSB))
- $40/16 = 2.5$, $0.5 \times 16 = 8 = 8$
- $2/16 = 0.125$, $0.125 \times 16 = 2 = 2$ (Most significant bit (MSB))
- Therefore, $650_{10} = 28A_{16}$

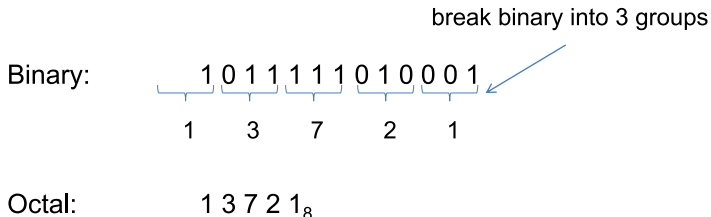
Counting in Octal

- Octal means eight
- In octal number system, a digit can take the value 0 through 7.

| Octal | Decimal | Binary |
|-------|---------|--------|
| 0 | 0 | 000 |
| 1 | 1 | 001 |
| 2 | 2 | 010 |
| 3 | 3 | 011 |
| 4 | 4 | 100 |
| 5 | 5 | 101 |
| 6 | 6 | 110 |
| 7 | 7 | 111 |

Octal → Decimal

- Octal numbers conversion: Convert 1011111010001_2 to octal



Octal Numbers

- Convert the following to binary
 - $25_8 = 10\ 101_2$
 - $140_8 = 001\ 100\ 000_2$
- Convert the following to octal
 - $110\ 101_2 = 65_8$
 - $1\ 101\ 111\ 001_2 = 1571_8$



<https://www.openlearning.com/courses/SKEE1223x>