Chapter 6
Karnaugh Maps
SKEE1223 Digital Electronics

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Overview

What is a Karnaugh Map?

- 2D truth table
- Certain logic simplifications can be easily recognized
- Resulting expression is guaranteed to be simplest
- Efficient up to 4 inputs vars
- 5-6 vars possible

2- and 3-variable Karnaugh Maps

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

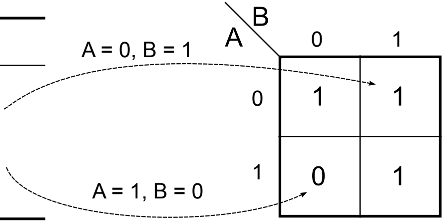
Figure: 2-variable K-map.

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Figure: 3-variable K-map.

2-var Truth Table → 2×2 Karnaugh Map

A	B	F
0	0	1
0	1	1
1	0	0
1	1	1



Truth table and K-map for $F(A,B) = \sum m(0,1,3)$.

3-var Truth Table $\rightarrow$ 4×2 Karnaugh Map

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

A \ BC	00	01	11	10
0	(0) 0	(1) 0	(3) 1	(2) 1
1	(4) 0	(5) 1	(7) 1	(6) 0

Example 1: Fill in K-map for prime numbers 0-7

Tip:

From specs to K-map: get truth table first.

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

A \ BC	00	01	11	10
0	(0) 0	(1) 0	(3) 1	(2) 1
1	(4) 0	(5) 1	(7) 1	(6) 0

Example 2: Fill in K-map for $F = AB + AC$

Tip:

From Boolean to K-map: expand to canonical SOP first.

$$\begin{aligned} F &= A\,B + A\,C \\ &= A\,B(C + \overline{C}) + A\,C(B + \overline{B}) \\ &= A\,B\,C + A\,B\,\overline{C} + A\,B\,C + A\,\overline{B}\,C \\ &= A\,\overline{B}\,C + A\,B\,\overline{C} + A\,B\,C \\ &= \sum m(5,6,7) \end{aligned}$$

A \ BC	00	01	11	10
0	(0) 0	(1) 0	(3) 1	(2) 0
1	(4) 0	(5) 1	(7) 1	(6) 0

Why K-maps Work?

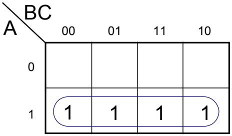
Unifying theorem:

$$A + \overline{A} = 1$$

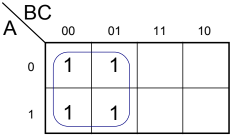
A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	0

$$\begin{aligned} A\bar{B}C + ABC &= AC(\bar{B} + B) \\ &= AC(1) \\ &= AC \end{aligned}$$

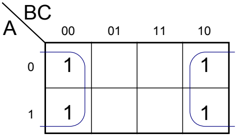
Loops in 3-variable Karnaugh Maps



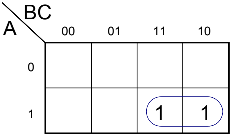
F = A



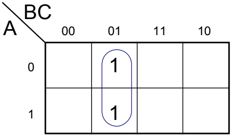
F = B



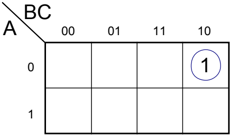
F = $\overline{C}$



F = AB



F = $\overline{B}C$



F = $\overline{A}B\overline{C}$

Simplification Rules

1. Loop or cover all minterms in square of powers of two.
2. If the minterms may be grouped more than one way, choose the biggest and simplest loop.
3. Minterms already covered may be reused.
4. The Boolean function is complete when all minterms are covered.

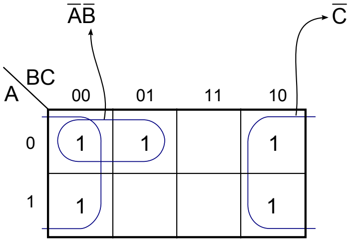
Example 3

Simplify $F = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$.

Solution:

Written in canonical SOP form, $F(A,B,C) = \sum m(0,1,2,4,6)$.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



4×4 Karnaugh Maps

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Figure: Type 1 K-map.

C

F_{0000}	F_{0001}	F_{0011}	F_{0010}
F_{0100}	F_{0101}	F_{0111}	F_{0110}
F_{1100}	F_{1101}	F_{1111}	F_{1110}
F_{1000}	F_{1001}	F_{1011}	F_{1010}

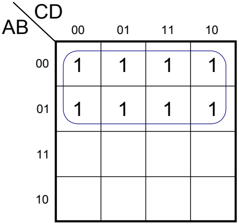
B

A

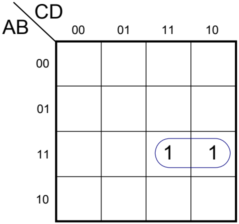
D

Figure: Type 2 K-map.

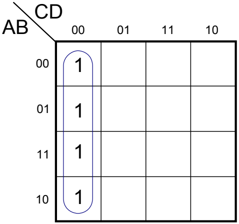
Loops in 4-variable Karnaugh Maps



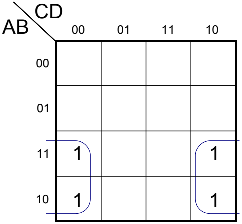
F = $\overline{A}$



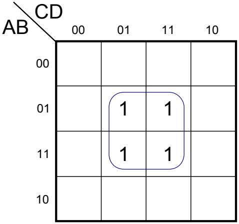
F = ABC



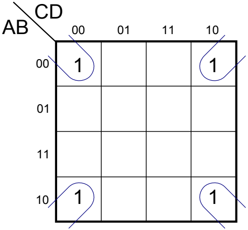
F = $\overline{C}\,\overline{D}$



F = $A\overline{D}$



F = BD



F = $\overline{B}\,\overline{D}$

Example 4

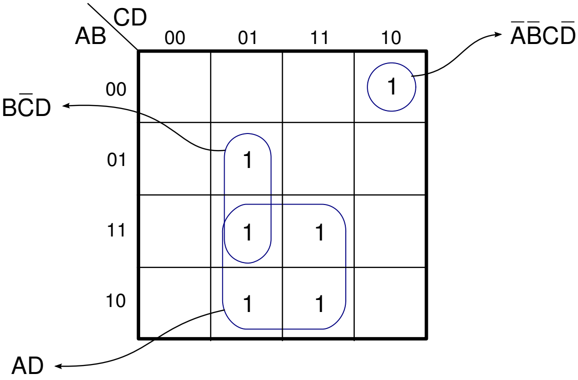


Figure: K-map for $F(A,B,C,D) = \sum m(3,5,9,11,13,15)$.

Rule 1: No Zeroes

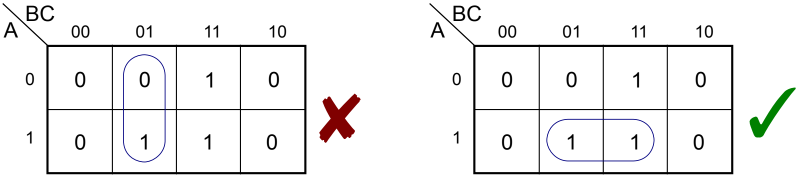


Figure: Groups may not include any cell containing a zero.

Rule 2: No Diagonals

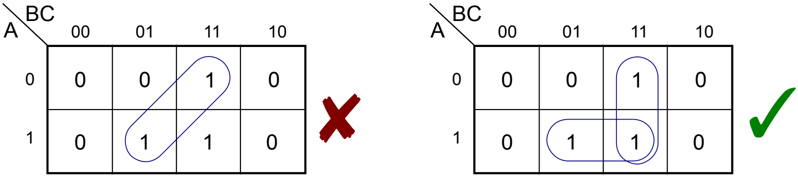


Figure: Groups may be horizontal or vertical, but not diagonal.

Rule 3: Only 2^n

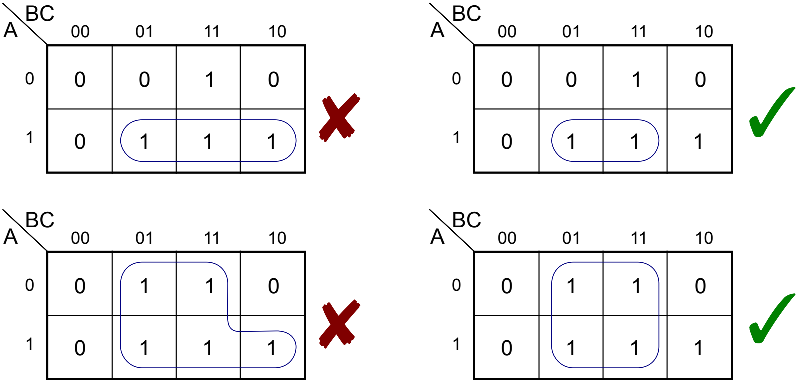


Figure: Groups must contain 1, 2, 4, 8, or in general 2^n cells.

Rule 4: Maximize Loop Size

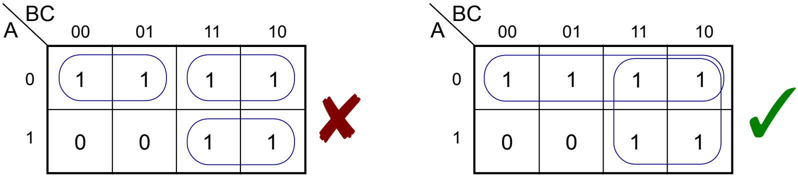


Figure: Each group should be as large as possible.

Rule 5: Must Loop All 1s

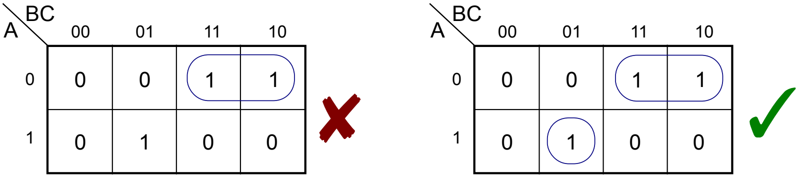


Figure: Every one must be in at least one group.

Rule 6: Groups May Overlap

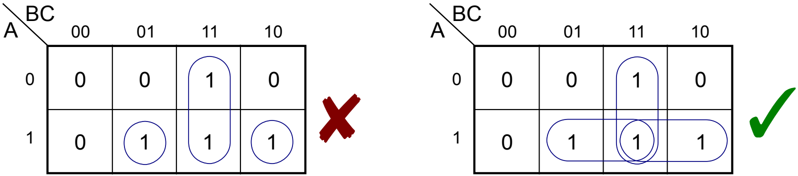


Figure: Groups may overlap.

Rule 7: Groups May Wrap Around

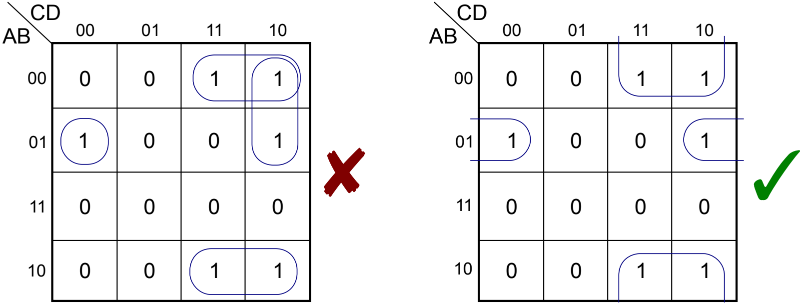


Figure: Groups may wrap around the table.

Rule 7: Minimize Number of Loops

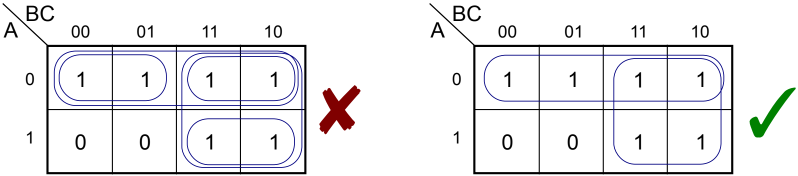
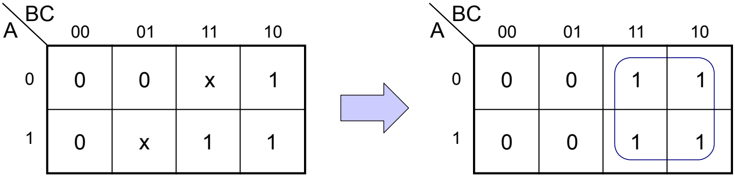


Figure: Fewest number of groups possible.

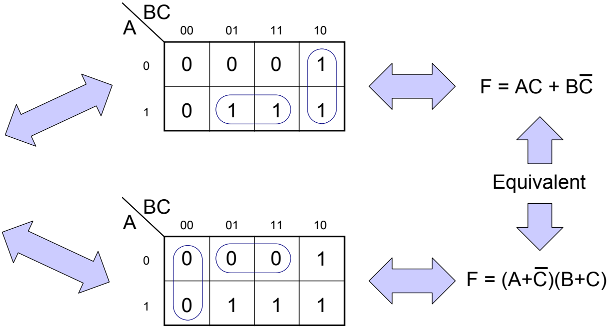
Don’t Care Karnaugh Maps

- Marked as $\times$
- $\times = 1$ to maximimize loop size
- $\times = 0$ to reduce number of loops



Karnaugh Maps for Maxterms

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



For More Info

1. https://www.openlearning.com/courses/SKEE1223x
2. http://www2.physics.umd.edu/~drew/spr07/KarnaughMaps-RulesofSimplification.htm