Chapter 6
Combinational Case Studies
SKEE2263 Digital Systems

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Design Prodecure

Step	Description
Specification	Write a specification for the circuit.
Formulation	Derive the truth table or initial Boolean equations that define the required relationships between inputs and outputs.
Optimization	Apply two-level and multi-level optimization.
Design entry	Draw the schematic for the resulting circuit or key in the hardware design language code.
Technology Mapping	Transform the logic diagram to a new diagram using the available implementation technology.
Verification	Verify the correctness of the final design.

The Problem with BCD

0110	=	6
+ 0011	=	+ 3
1001	=	9

Correct:
Result is in BCD

0101	=	5
+ 0111	=	+ 7
1100	=	12

Wrong:
Result is not in BCD

Correction Factor (when sum>9)

	0101	5
	+ 0111	+ 7
Non BCD	1100	12
Adjustment	+ 0110	+ 6
Correct BCD	1 0010	1 2

	1000	8
	+ 0111	+ 7
Non BCD	1111	15
Adjustment	+ 0110	+ 6
Correct BCD	1 0101	1 5

	1001	9
	+ 1001	+ 9
Non BCD	1 0010	18
Adjustment	+ 0110	+ 6
Correct BCD	1 1000	1 8

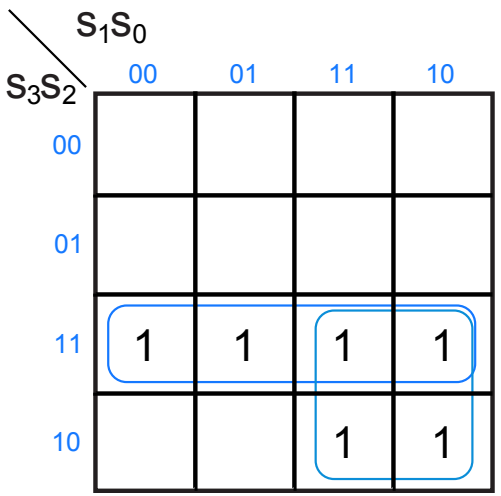
All Possible Adder Outputs

Decimal	Binary Sum					BCD Sum				
	C	s_3	s_2	s_1	s_0	K	z_3	z_2	z_1	z_0
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	1
2	0	0	0	1	0	0	0	0	1	0
3	0	0	0	1	1	0	0	0	1	1
4	0	0	1	0	0	0	0	1	0	0
5	0	0	1	0	1	0	0	1	0	1
6	0	0	1	1	0	0	0	1	1	0
7	0	0	1	1	1	0	0	1	1	1
8	0	1	0	0	0	0	1	0	0	0
9	0	1	0	0	1	0	1	0	0	1
10	0	1	0	1	0	1	0	0	0	0
11	0	1	0	1	1	1	0	0	0	1
12	0	1	1	0	0	1	0	0	1	0
13	0	1	1	0	1	1	0	0	1	1
14	0	1	1	1	0	1	0	1	0	0
15	0	1	1	1	1	1	0	1	0	1
16	1	0	0	0	0	1	0	1	1	0
17	1	0	0	0	1	1	0	1	1	1
18	1	0	0	1	0	1	1	0	0	0
19	1	0	0	1	1	1	1	0	0	1

Correction Factor

When the sum exceeds 9:

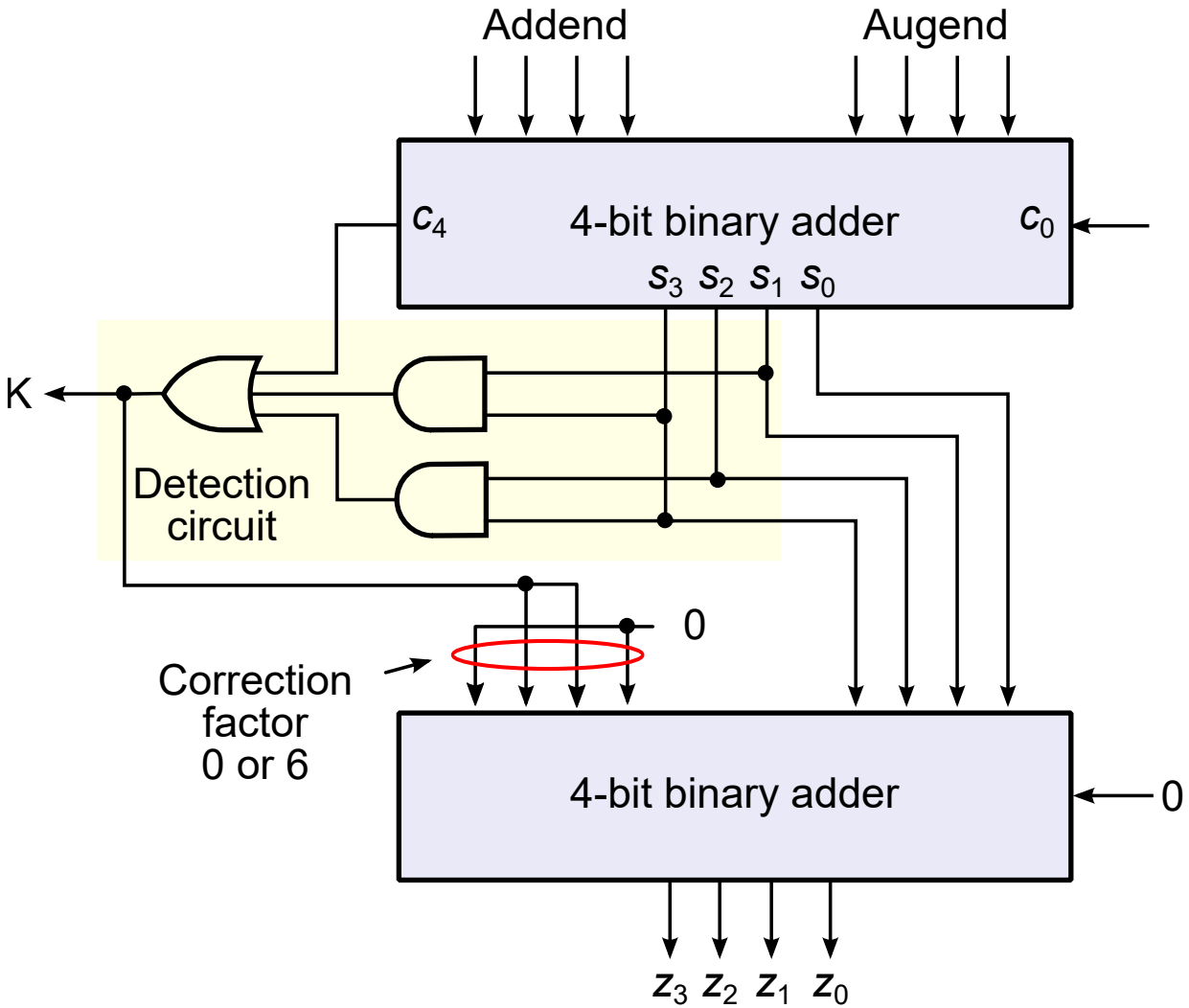
- Carry bit C is 1, or
- The binary sum is between 1010 and 1111



The correction step (adding 0110) is performed if C or f is true:

$$K = C + s_3 s_2 + s_3 s_1$$

BCD Adder Circuit



Double-dabble algorithm

For an n-bit binary number, the steps are:

1. Find the number of bits which holds the biggest BCD output.
2. Divide the output into BCD columns of 4 bits each
3. Shift the binary number one bit
4. If the binary number in any of the BCD columns is 5 or greater, add 3 to the BCD columns.
5. Repeat from Step 3 until all n bits have been shifted.

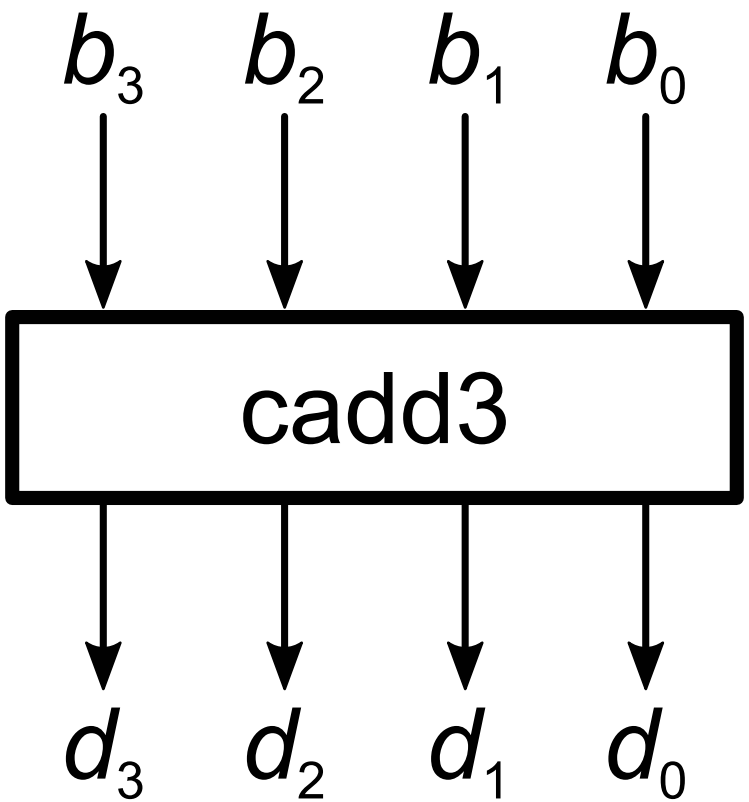
3F $\rightarrow$ 63

Operation	Tens	Ones	Binary
HEX			3F
Start			111111
Shift 1		1	11111
Shift 2		11	1111
Shift 3		111	111
Add 3		1010	111
Shift 4	1	0101	11
Add 3	1	1000	11
Shift 5	11	0001	1
Shift 6	110	0011	
BCD	6	3	

FF $\rightarrow$ 255

Operation	Hundreds	Tens	Ones	Binary	
Start				1111	1111
Shift 1			1	1111	111
Shift 2			11	1111	11
Shift 3			111	1111	1
Add 3			1010	1111	1
Shift 4		1	0101	1111	
Add 3		1	1000	1111	
Shift 5		11	0001	111	
Shift 6		110	0011	11	
Add 3		1001	0011	11	
Shift 7	1	0010	0111	1	
Add 3	1	0010	1010	1	
Shift 8	10	0101	0101		

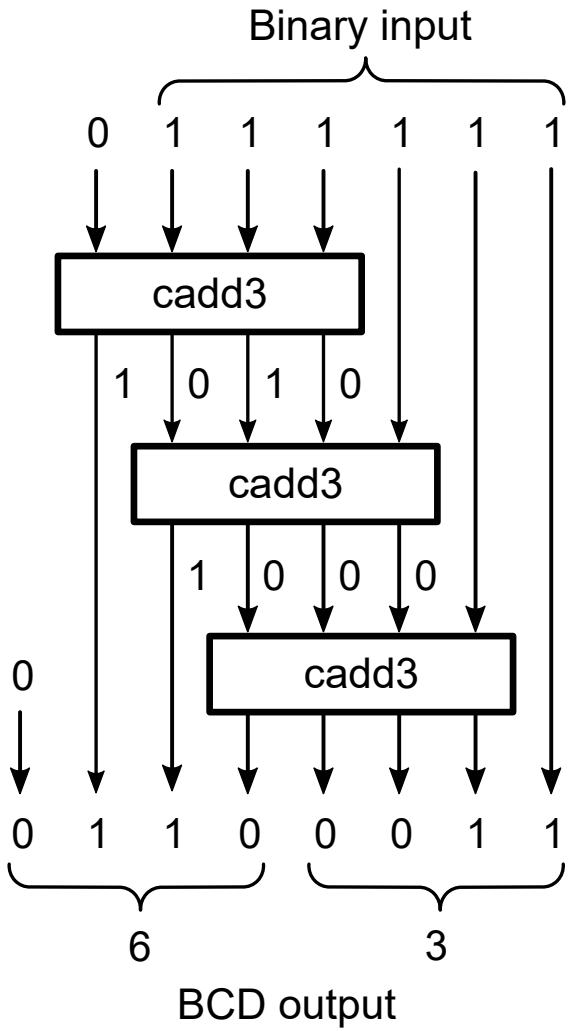
add3 Module



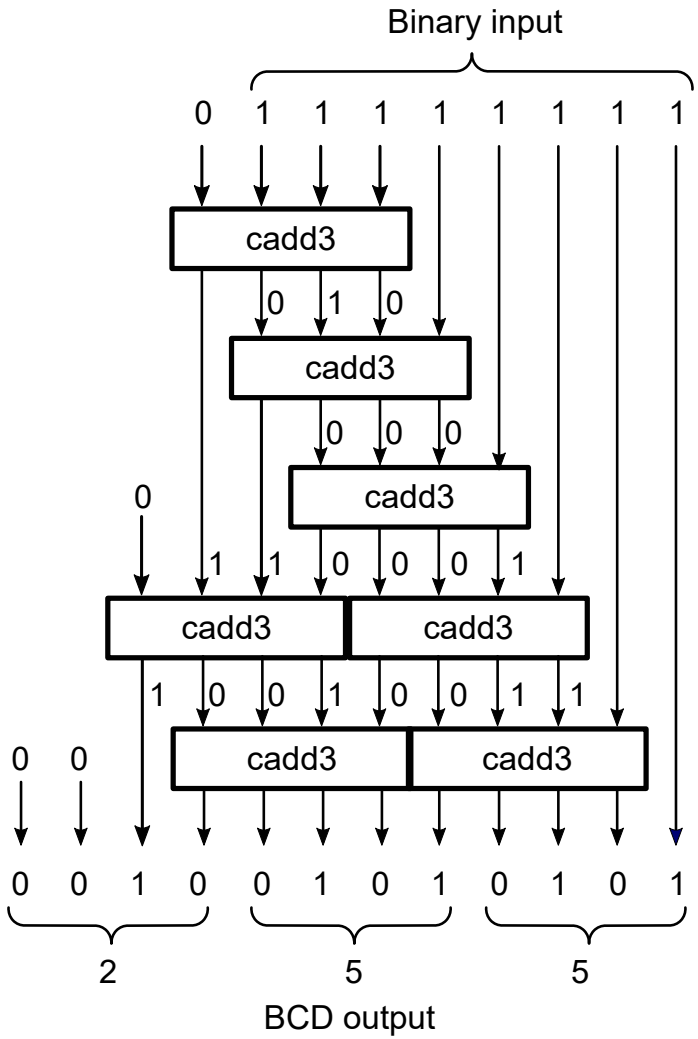
add3 Module

Binary Input				Decimal Output				Function
b_3	b_2	b_1	b_0	d_3	d_2	d_1	d_0	
0	0	0	0	0	0	0	0	Add 0
0	0	0	1	0	0	0	1	
0	0	1	0	0	0	1	0	
0	0	1	1	0	0	1	1	
0	1	0	0	0	1	0	0	
0	1	0	1	1	0	0	0	Add 3
0	1	1	0	1	0	0	1	
0	1	1	1	1	0	1	0	
1	0	0	0	1	0	1	1	
1	0	0	1	1	1	0	0	
1	0	1	0	x	x	x	x	Don't care
1	0	1	1	x	x	x	x	
1	1	0	0	x	x	x	x	
1	1	0	1	x	x	x	x	
1	1	1	0	x	x	x	x	
1	1	1	1	x	x	x	x	

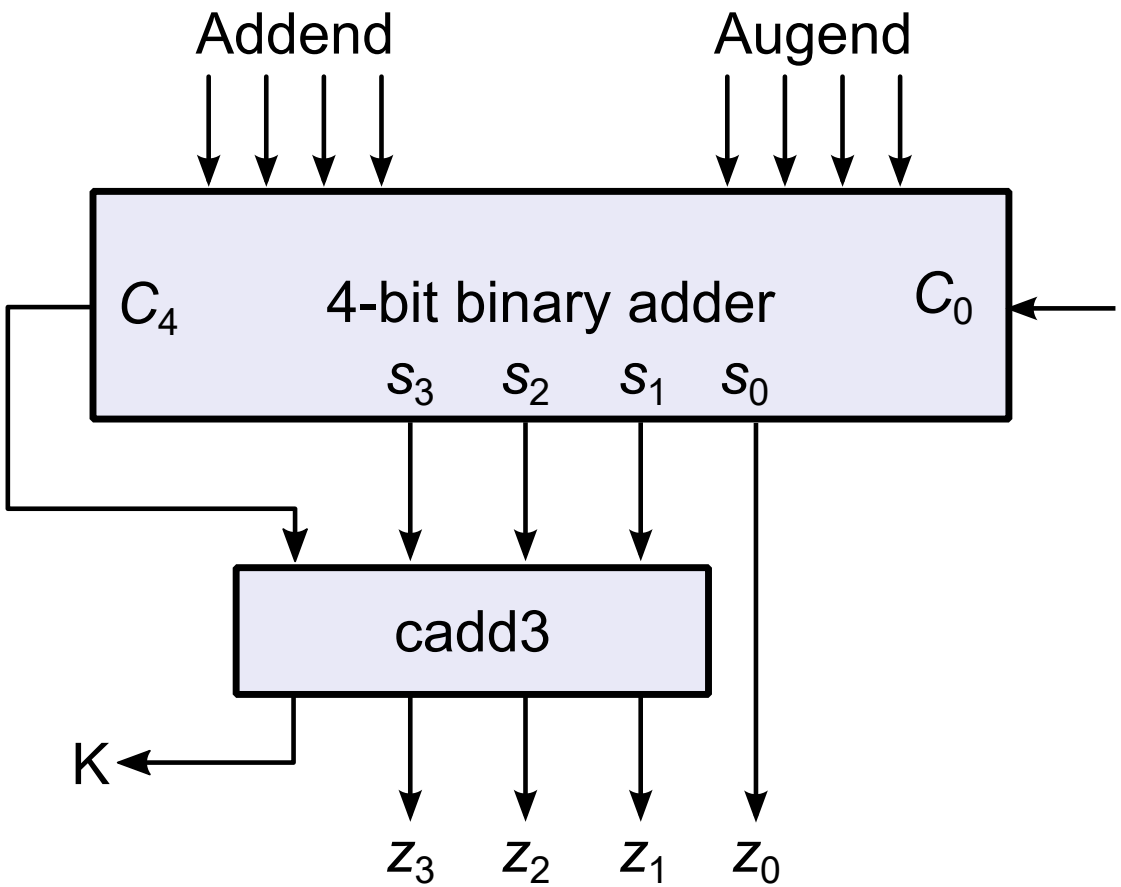
6 bit binary $\rightarrow$ 2 BCD digits



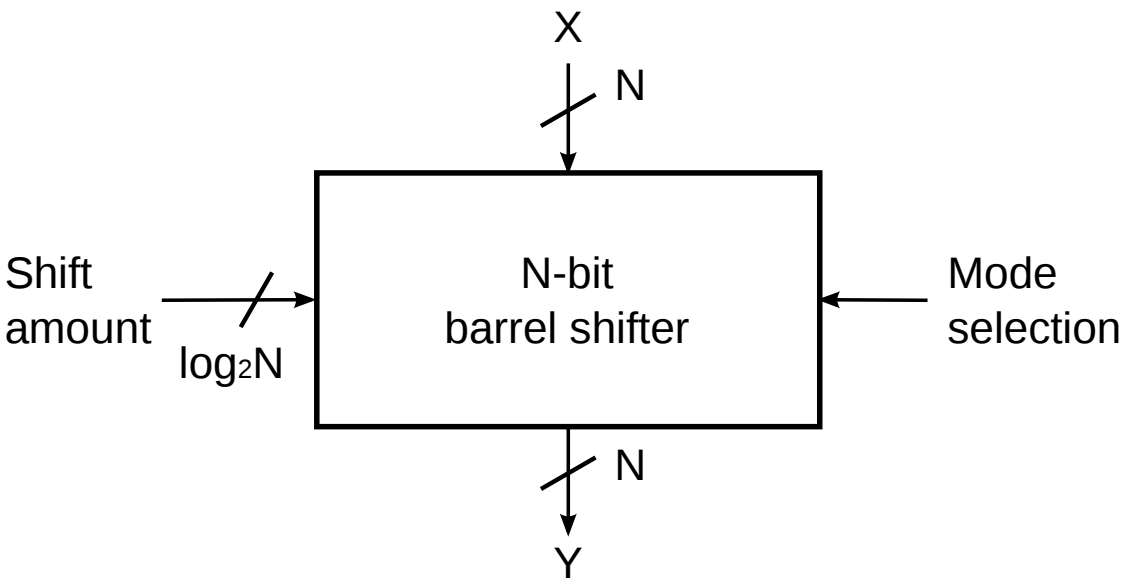
8 bit binary $\rightarrow$ 3 BCD digits



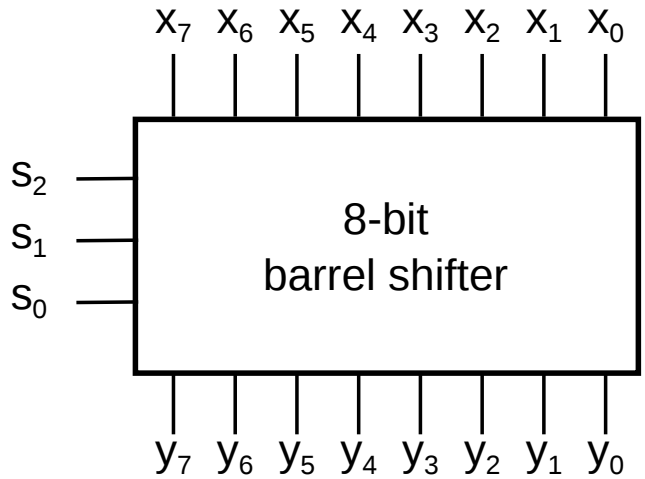
Improved BCD Adder



Generic Barrel Shifter

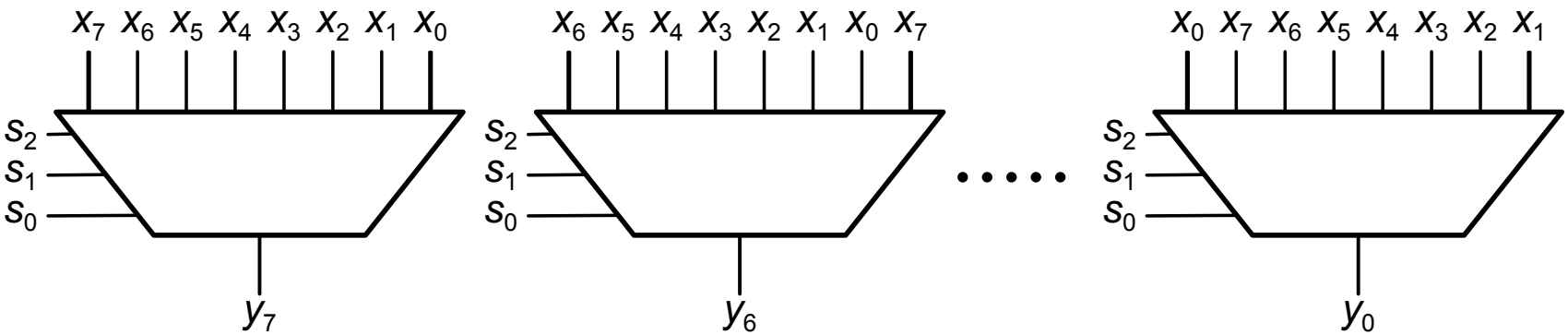


Barrel Shifter Outputs

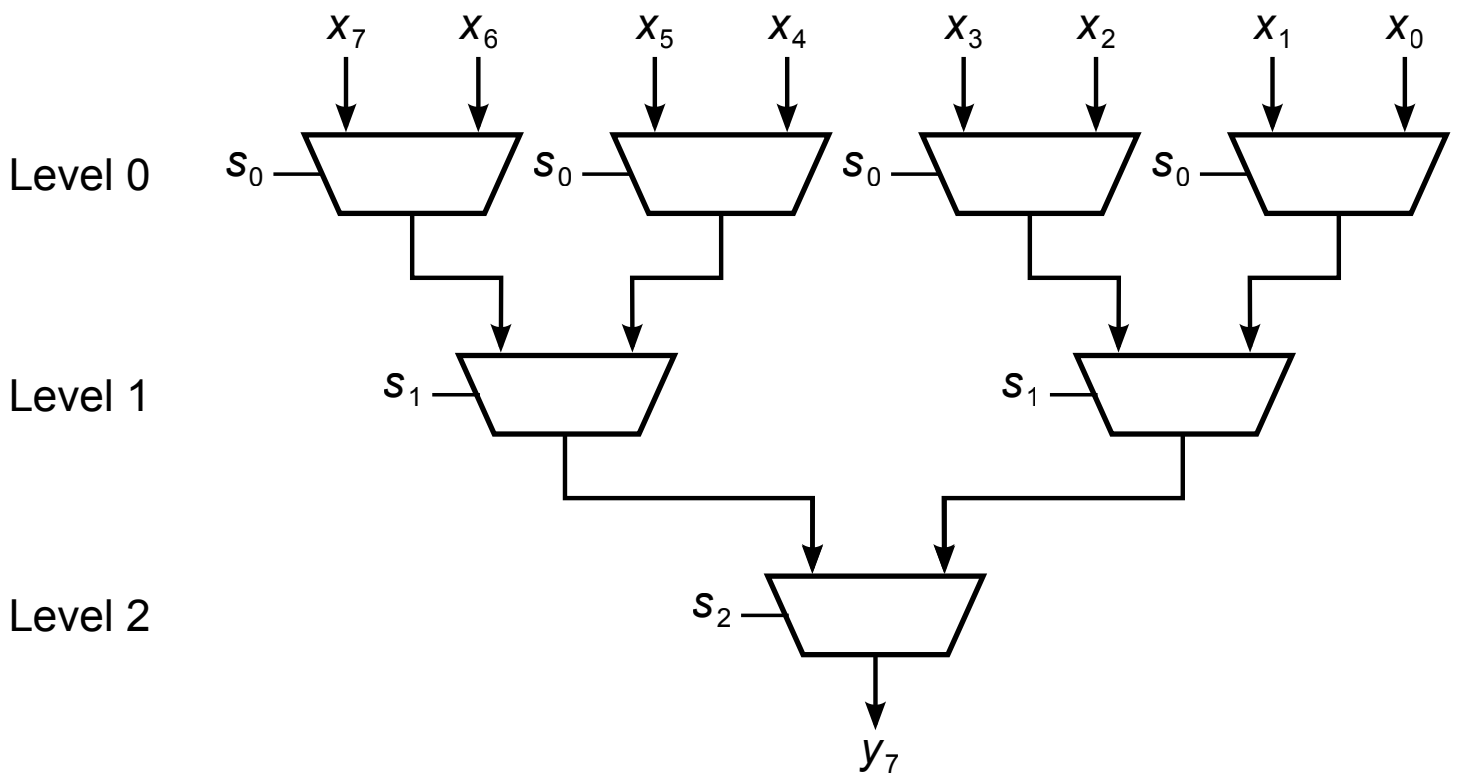


Shift amount	y_7	y_6	y_5	y_4	y_3	y_2	y_1	y_0
0	x_7	x_6	x_5	x_4	x_3	x_2	x_1	x_0
1	x_6	x_5	x_4	x_3	x_2	x_1	x_0	x_7
2	x_5	x_4	x_3	x_2	x_1	x_0	x_7	x_6
4	x_3	x_2	x_1	x_0	x_7	x_6	x_5	x_4
7	x_0	x_7	x_6	x_5	x_4	x_3	x_2	x_1

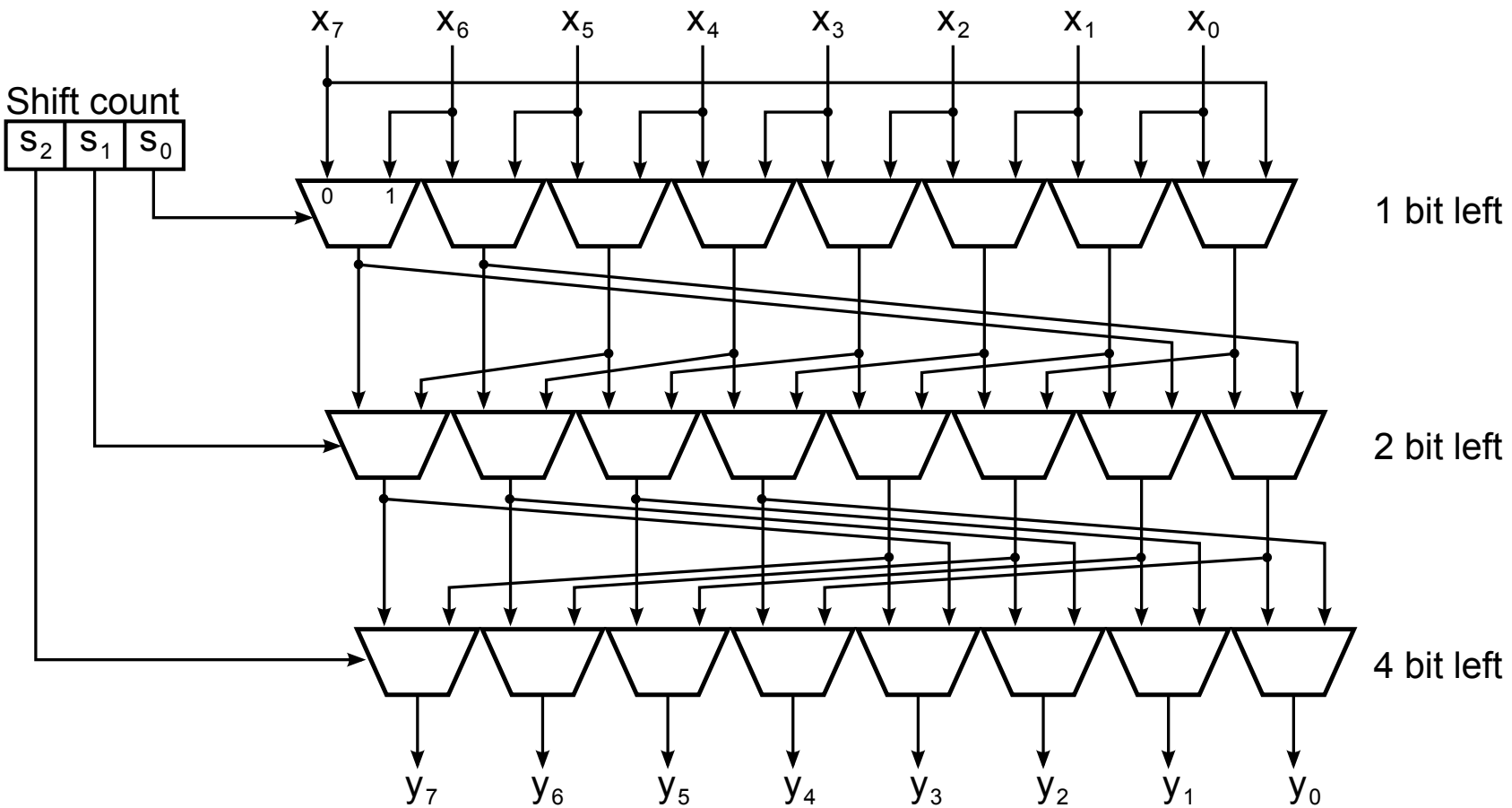
Implementation by 8:1 MUX



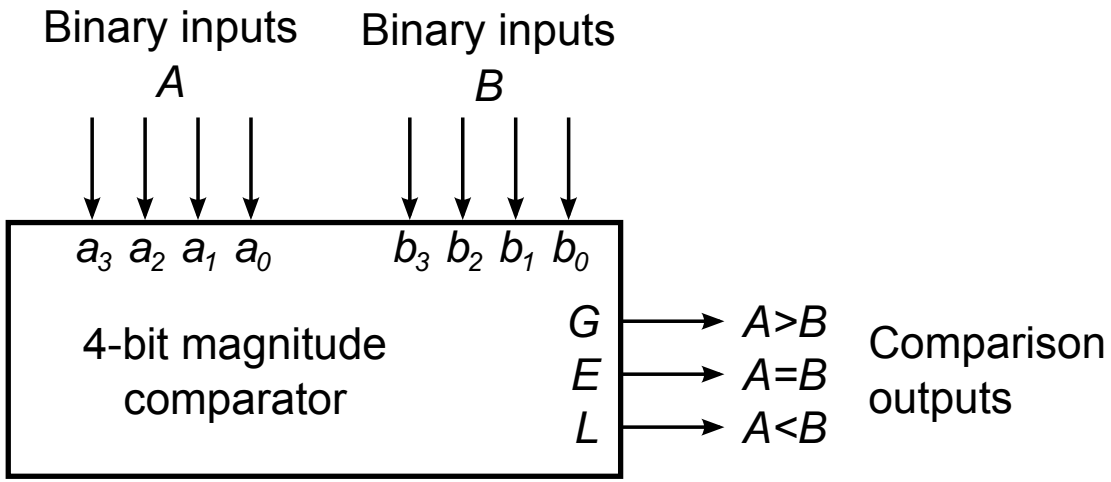
Implementation by 2:1 MUX



Eight-bit Barrel Shifter



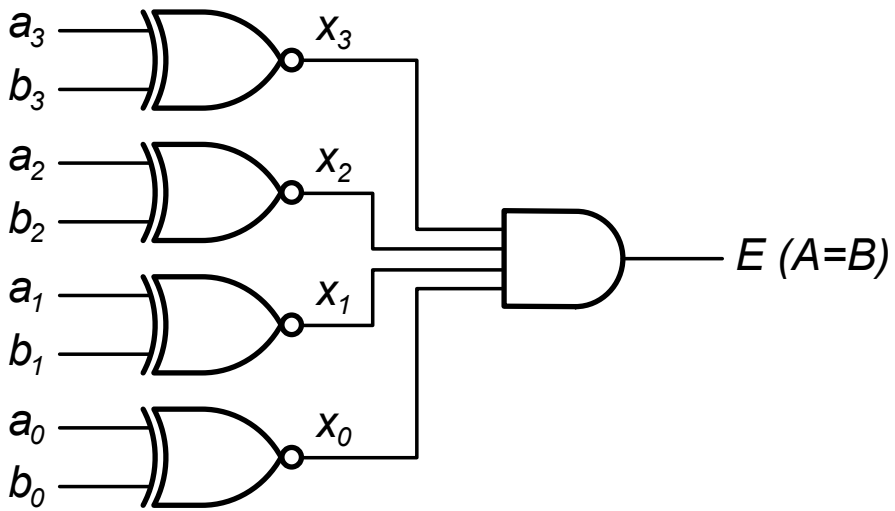
Magnitude Comparator Top-Level



$$E = (A = B) = x_3 x_2 x_1 x_0$$
$$L = (A < B) = a_3' b_3 + x_3 a_2' b_2 + x_3 x_2 a_1' b_1 + x_3 x_2 x_1 a_0' b_0$$
$$G = (A > B) = a_3 b_3' + x_3 a_2 b_2' + x_3 x_2 a_1 b_1' + x_3 x_2 x_1 a_0 b_0'$$

where $x_i = a_i \odot b_i = a_i b_i + a_i' b_i'$ for $i = 0, 1, 2, 3$.

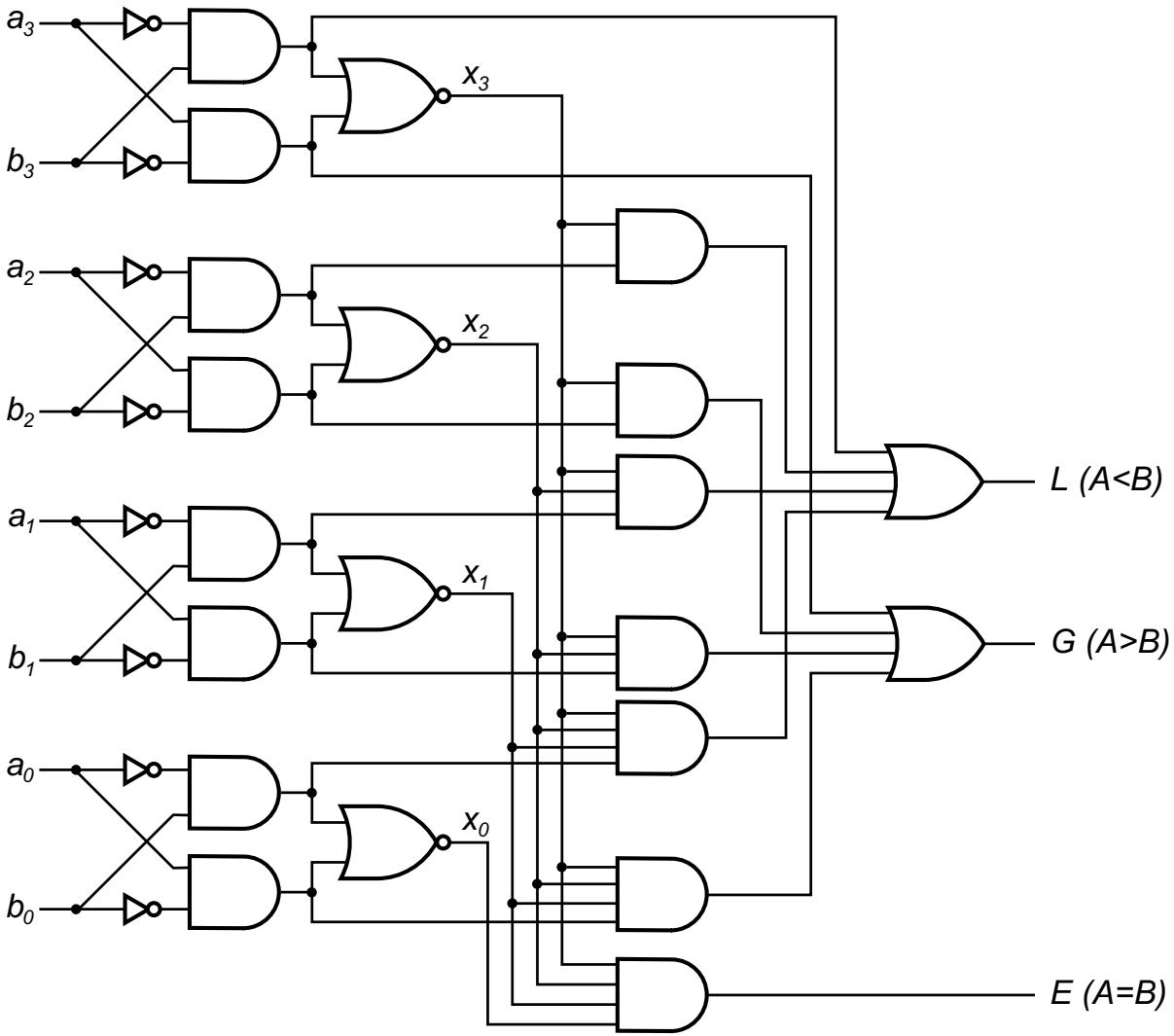
4-Bit Equality Comparator



$$E = (A = B) = x_3 x_2 x_1 x_0$$

where $x_i = a_i \odot b_i = a_i b_i + a_i' b_i'$

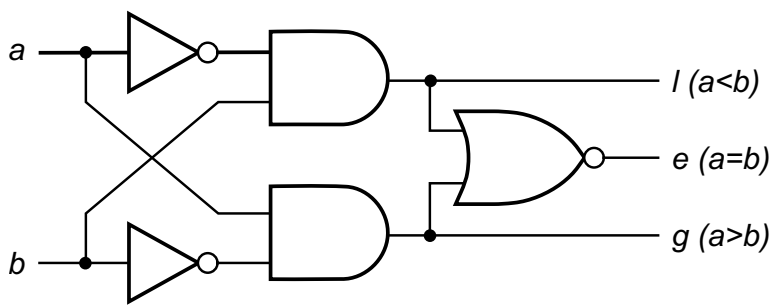
4-Bit Comparator



Comparator Slice: End Module

a	b	l $(a < b)$	e $(a = b)$	g $(a > b)$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

$$l = (a < b) = a'b$$
$$e = (a = b) = a'b' + ab = a \odot b$$
$$g = (a > b) = ab'$$

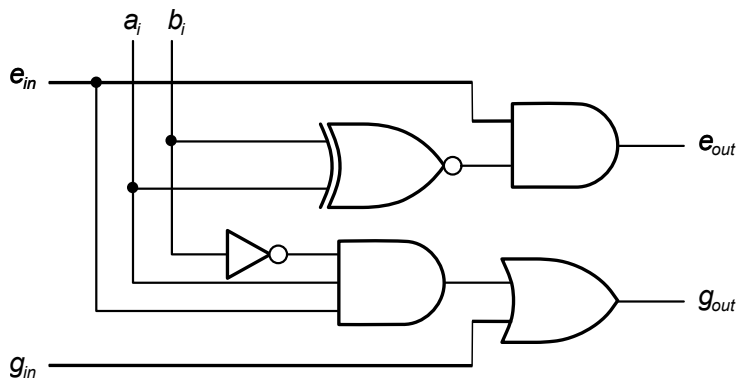


Comparator Slice: Cascadable Module

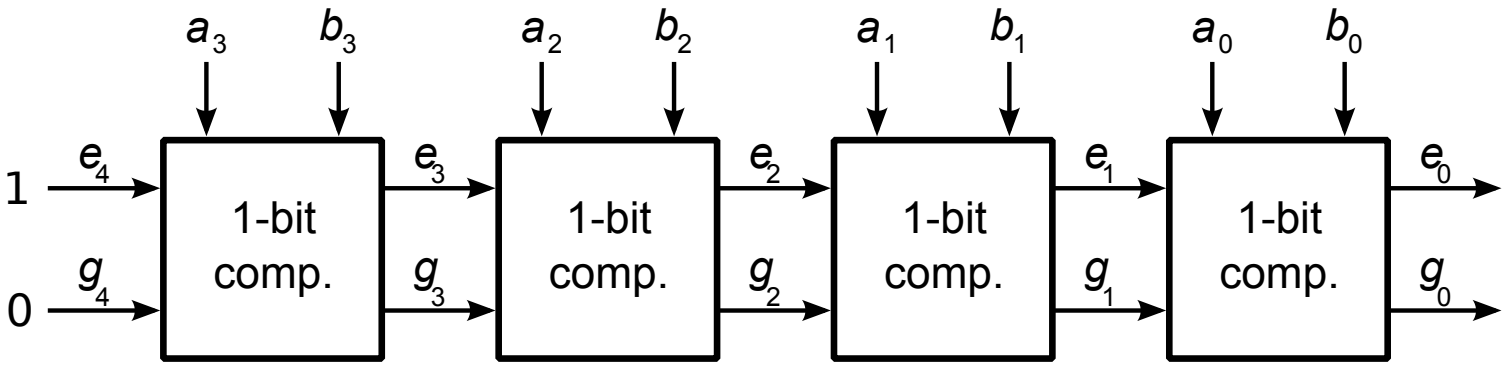
e_{in}	g_{in}	a_i	b_i	e_{out}	g_{out}
0	0	×	×	0	0
0	1	×	×	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0

$$e_{out} = e_{in} \cdot (a_i \odot b_i)$$

$$g_{out} = g_{in} + e_{in} \cdot a_i \cdot b_i'$$



4-Bit Iterative Comparator



4-Bit Incrementer