Chapter 11
FSM Reverse Engineering
SKEE2263 Digital Systems

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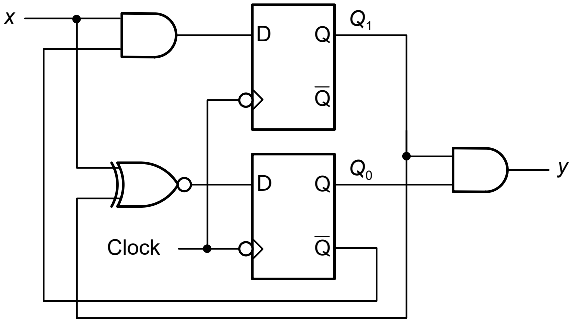
Reverse Analysis / Analysis

What is Reverse Engineering?

Work in reverse direction to derive the state diagram from a given circuit.

- Determine circuit model - Moore or Mealy
- Find:
  1. Number of states
  2. Next state and output equations
  3. Next state and and output K-maps
- Get State table
- Draw the state diagram

Determine Machine Type



- Moore or Mealy?

  $\Rightarrow$ AND inputs connects only to FF outputs

  $\Rightarrow$ output depends only on present state

  $\therefore$ this is a Moore machine

Circuit → Equations

1. #FF: 2

   ⇒ Number of states = 4

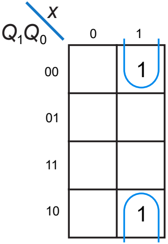
2. Next state equations

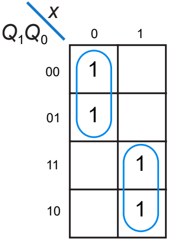
$$Q_1{}^+ = D_1 = Q_0'x$$
$$Q_0{}^+ = D_0 = \overline{Q_1 \oplus x} = Q_1x + Q_1'x'$$

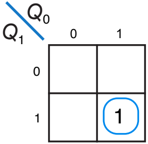
3. Output equation

$$y = Q_1Q_0$$

Equations → K-maps → minterms







$$Q_1^+ = Q_0' x$$
$$= \Sigma(1, 5)$$

$$Q_0^+ = Q_1 x + Q_1' x'$$
$$= \Sigma(0, 2, 5, 7)$$

$$y = Q_1 Q_0$$

Alternative: Equations → Canonical SOP minterms

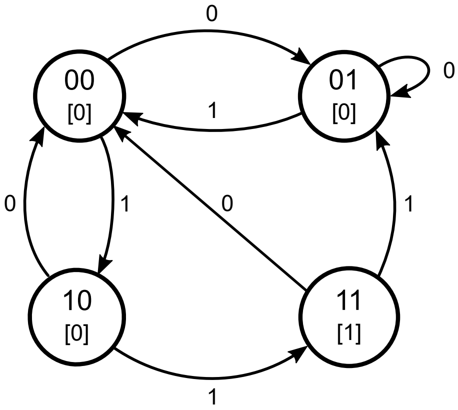
$$\begin{aligned}{Q_1}^+ &= Q_0'x \\ &= Q_0'x(Q_1 + Q_1') \\ &= Q_1'Q_0'x + Q_1Q_0'x \\ &= \Sigma(1,5)\end{aligned}$$

$$\begin{aligned}{Q_0}^+ &= \overline{Q_1 \oplus x} = Q_1x + Q_1'x' \\ &= Q_1x(Q_0' + Q_0) + Q_1'x'(Q_0' + Q_0) \\ &= Q_1Q_0'x + Q_1Q_0x + Q_1'Q_0'x' + Q_1'Q_0x' \\ &= \Sigma(0,2,5,7)\end{aligned}$$

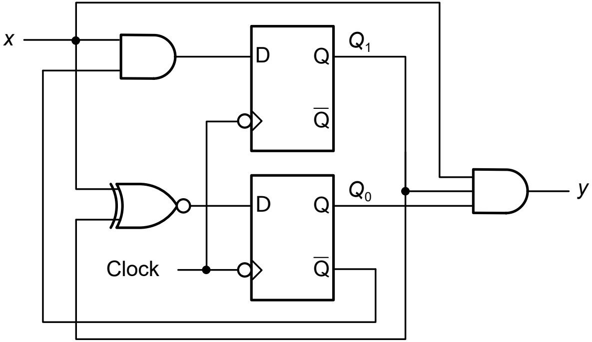
Minterms → State Table

Present State	**Input**	**Next State**	**Output**
Q_1Q_0	x	$Q_1^+Q_0^+$	y
00	0 1	01 10	0
01	0 1	01 00	0
10	0 1	00 11	0
11	0 1	00 01	1

State Table → State Diagram



Determine Machine Type



- Moore or Mealy?
  - ⇒ Output logic gets inputs from FF and x (system input)
  - ⇒ output depends on present state AND present input
  - ∴ this is a Mealy machine

Circuit $\rightarrow$ Equations

1. #FF: 2
   $\Rightarrow$ Number of states = 4
2. Next state equations

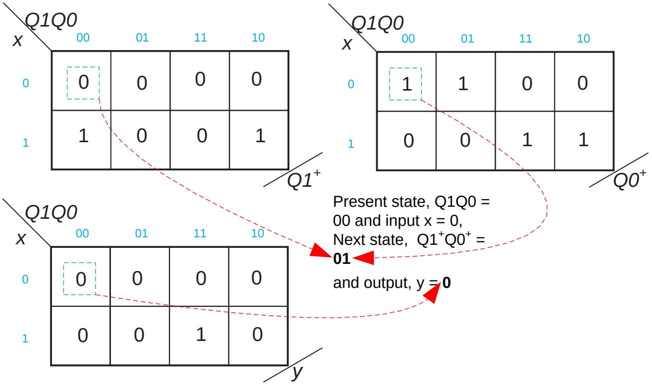
$$Q_1{}^{+} = D_1 = Q_0' x$$

$$Q_0{}^{+} = D_0 = \overline{Q_1 \oplus x} = Q_1 x + Q_1' x'$$

3. Output equation

$$y = Q_1 Q_0 x$$

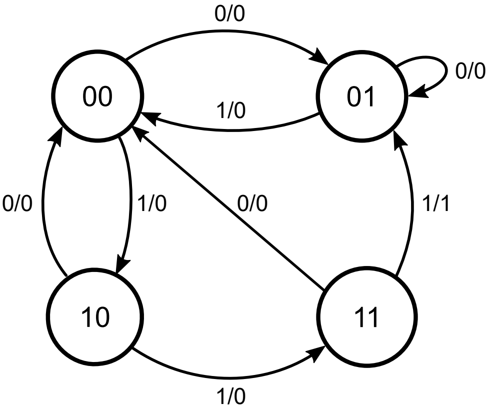
Equations → K-maps



Trick

Derive state diagram directly from K-maps

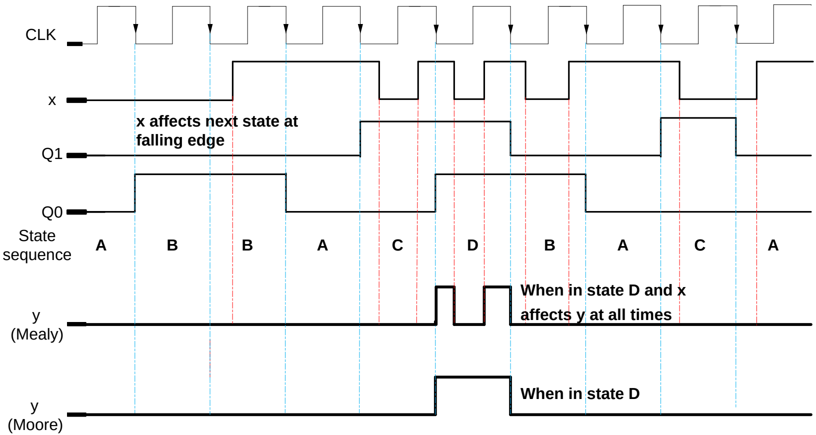
K-maps → State Diagram



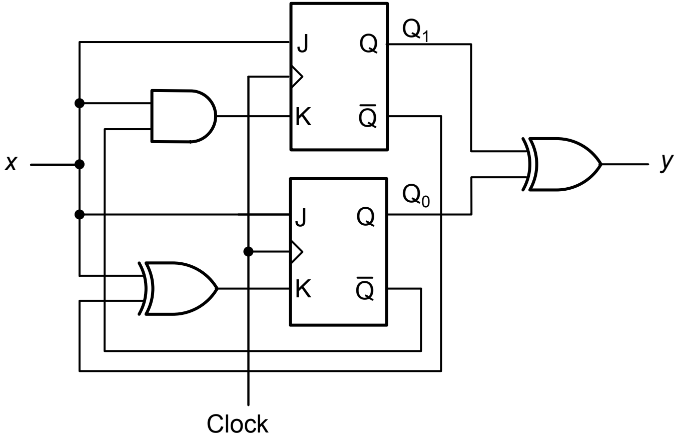
State Table for Reference

Present State	Input	Next State	Output
Q_1Q_0	x	$Q_1^+Q_0^+$	y
00	0	01	0
	1	10	0
01	0	01	0
	1	00	0
10	0	00	0
	1	11	0
11	0	00	0
	1	01	1

Waveforms of Both Moore & Mealy Machines



Determine Machine Type



- This is a Moore machine

Circuit → FF Input Equations

Flip-Flop input equations:

$$J_1 = x \qquad J_0 = x$$

$$K_1 = Q_0'x \qquad K_0 = Q_1 \oplus x$$

Output equation:

$$y = Q_1 \oplus Q_0$$

FF Input Eqs → Next State Eqs (EXTRA STEP)

JKFF characteristic equations:

$$JQ' + K'Q$$

Substituting:

$$\begin{aligned} Q_1^+ &= J_1 Q_1' + K_1' Q_1 \\ &= xQ_1' + [Q_0' x]' Q_1 \\ &= Q_1' x + [Q_0'' + x'] Q_1 \\ &= Q_1' x + Q_1 Q_0 + Q_1 x' \\ &= \Sigma m(1,3,4,6,7) \end{aligned}$$

$$\begin{aligned} Q_0^+ &= J_0 Q_0' + K_0' Q_0 \\ &= xQ_0' + [Q_1' \oplus x]' Q_0 \\ &= Q_0' x + Q_0 [Q_1' x + Q_1 x'] \\ &= Q_0' x + Q_1' Q_0 x + Q_1 Q_0 x' \\ &= \Sigma m(1,3,5,6) \end{aligned}$$

Minterms → State Table

Present State	Input	Next State	Output
Q_1Q_0	x	$Q_1^+Q_0^+$	y
00	0	00	0
	1	11	
01	0	00	1
	1	11	
10	0	10	1
	1	01	
11	0	11	0
	1	10	

State Table → State Diagram